

slope, the most convenient reference frame for analyzing her motion is one with the  $x$ -axis along and the  $y$ -axis perpendicular to the incline. In this frame, both the normal and the frictional forces lie along coordinate axes, the components of the weight are  $mg \sin \theta$  along the slope and  $mg \cos \theta$  at right angles into the slope, and the only acceleration is along the  $x$ -axis ( $a_y = 0$ ).

### Solution

We can now apply Newton's second law to the snowboarder:

$$\begin{aligned}\sum F_x &= ma_x & \sum F_y &= ma_y \\ mg \sin \theta - \mu_k N &= ma_x & N - mg \cos \theta &= m(0).\end{aligned}$$

From the second equation,  $N = mg \cos \theta$ . Upon substituting this into the first equation, we find

$$\begin{aligned}a_x &= g(\sin \theta - \mu_k \cos \theta) \\ &= g(\sin 13^\circ - 0.20 \cos 13^\circ) = 0.29 \text{ m/s}^2.\end{aligned}$$

### Significance

Notice from this equation that if  $\theta$  is small enough or  $\mu_k$  is large enough,  $a_x$  is negative, that is, the snowboarder slows down.



**6.8 Check Your Understanding** The snowboarder is now moving down a hill with incline  $10.0^\circ$ . What is the skier's acceleration?

## 6.3 | Centripetal Force

### Learning Objectives

By the end of the section, you will be able to:

- Explain the equation for centripetal acceleration
- Apply Newton's second law to develop the equation for centripetal force
- Use circular motion concepts in solving problems involving Newton's laws of motion

In **Motion in Two and Three Dimensions**, we examined the basic concepts of circular motion. An object undergoing circular motion, like one of the race cars shown at the beginning of this chapter, must be accelerating because it is changing the direction of its velocity. We proved that this centrally directed acceleration, called centripetal acceleration, is given by the formula

$$a_c = \frac{v^2}{r}$$

where  $v$  is the velocity of the object, directed along a tangent line to the curve at any instant. If we know the angular velocity  $\omega$ , then we can use

$$a_c = r\omega^2.$$

Angular velocity gives the rate at which the object is turning through the curve, in units of rad/s. This acceleration acts along the radius of the curved path and is thus also referred to as a radial acceleration.

An acceleration must be produced by a force. Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge. Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law

of motion, net force is mass times acceleration:  $F_{\text{net}} = ma$ . For uniform circular motion, the acceleration is the centripetal acceleration:  $a = a_c$ . Thus, the magnitude of centripetal force  $F_c$  is

$$F_c = ma_c.$$

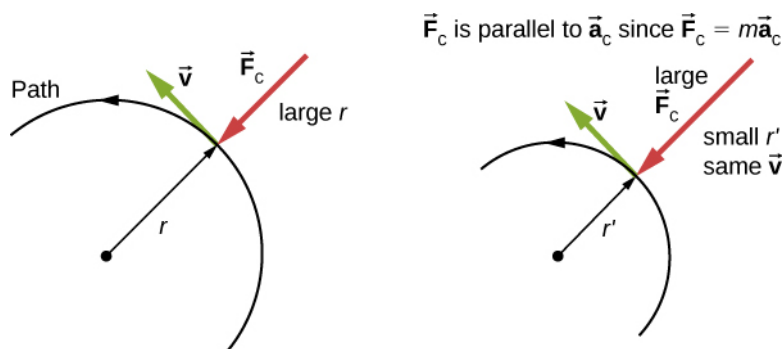
By substituting the expressions for centripetal acceleration  $a_c$  ( $a_c = \frac{v^2}{r}$ ;  $a_c = r\omega^2$ ), we get two expressions for the centripetal force  $F_c$  in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r}; \quad F_c = mr\omega^2. \quad (6.3)$$

You may use whichever expression for centripetal force is more convenient. Centripetal force  $\vec{F}_c$  is always perpendicular to the path and points to the center of curvature, because  $\vec{a}_c$  is perpendicular to the velocity and points to the center of curvature. Note that if you solve the first expression for  $r$ , you get

$$r = \frac{mv^2}{F_c}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve, as in **Figure 6.20**.

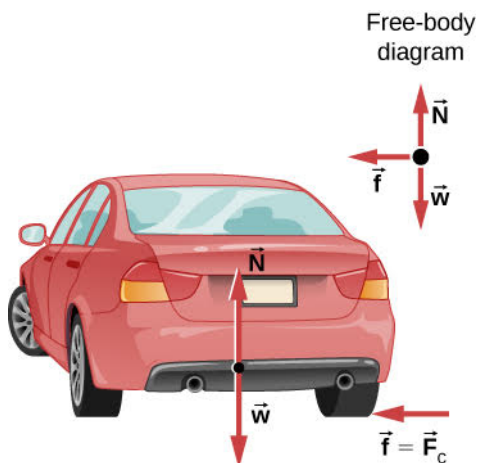


**Figure 6.20** The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the  $F_c$ , the smaller the radius of curvature  $r$  and the sharper the curve. The second curve has the same  $v$ , but a larger  $F_c$  produces a smaller  $r'$ .

### Example 6.15

#### What Coefficient of Friction Do Cars Need on a Flat Curve?

- Calculate the centripetal force exerted on a 900.0-kg car that negotiates a 500.0-m radius curve at 25.00 m/s.
- Assuming an unbanked curve, find the minimum static coefficient of friction between the tires and the road, static friction being the reason that keeps the car from slipping (**Figure 6.21**).



**Figure 6.21** This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

### Strategy

- a. We know that  $F_c = \frac{mv^2}{r}$ . Thus,

$$F_c = \frac{mv^2}{r} = \frac{(900.0 \text{ kg})(25.00 \text{ m/s})^2}{(500.0 \text{ m})} = 1125 \text{ N}.$$

- b. **Figure 6.21** shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is  $\mu_s N$ , where  $\mu_s$  is the static coefficient of friction and  $N$  is the normal force. The normal force equals the car's weight on level ground, so  $N = mg$ . Thus the centripetal force in this situation is

$$F_c \equiv f = \mu_s N = \mu_s mg.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the equation

$$F_c = m \frac{v^2}{r},$$

we obtain

$$m \frac{v^2}{r} = \mu_s mg.$$

We solve this for  $\mu_s$ , noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}.$$

Substituting the knowns,

$$\mu_s = \frac{(25.00 \text{ m/s})^2}{(500.0 \text{ m})(9.80 \text{ m/s}^2)} = 0.13.$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

### Significance

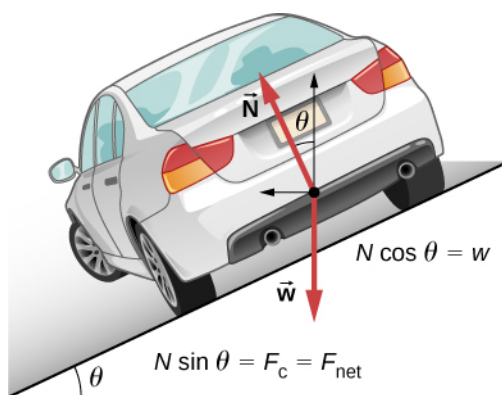
The coefficient of friction found in **Figure 6.21(b)** is much smaller than is typically found between tires and roads. The car still negotiates the curve if the coefficient is greater than 0.13, because static friction is a responsive force, able to assume a value less than but no more than  $\mu_s N$ . A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that, in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less, as discussed next.



**6.9 Check Your Understanding** A car moving at 96.8 km/h travels around a circular curve of radius 182.9 m on a flat country road. What must be the minimum coefficient of static friction to keep the car from slipping?

## Banked Curves

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve (**Figure 6.22**). The greater the angle  $\theta$ , the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an “ideally banked curve,” the angle  $\theta$  is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for  $\theta$  for an ideally banked curve and consider an example related to it.



**Figure 6.22** The car on this banked curve is moving away and turning to the left.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force  $N$  in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

**Figure 6.22** shows a free-body diagram for a car on a frictionless banked curve. If the angle  $\theta$  is ideal for the speed and radius, then the net external force equals the necessary centripetal force. The only two external forces acting on the car are its weight  $\vec{w}$  and the normal force of the road  $\vec{N}$ . (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude  $mv^2/r$ . Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, so this must equal the centripetal force, that is,

$$N \sin \theta = \frac{mv^2}{r}.$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From **Figure 6.22**, we see that the vertical component of the normal force is  $N \cos \theta$ , and the only other vertical force is the car's weight. These

must be equal in magnitude; thus,

$$N \cos \theta = mg.$$

Now we can combine these two equations to eliminate  $N$  and get an expression for  $\theta$ , as desired. Solving the second equation for  $N = mg/(\cos\theta)$  and substituting this into the first yields

$$mg \frac{\sin \theta}{\cos \theta} = \frac{mv^2}{r}$$

$$mg \tan \theta = \frac{mv^2}{r}$$

$$\tan \theta = \frac{v^2}{rg}.$$

Taking the inverse tangent gives

$$\theta = \tan^{-1} \left( \frac{v^2}{rg} \right). \quad (6.4)$$

This expression can be understood by considering how  $\theta$  depends on  $v$  and  $r$ . A large  $\theta$  is obtained for a large  $v$  and a small  $r$ . That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve were frictionless. Note that  $\theta$  does not depend on the mass of the vehicle.

## Example 6.16

### What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100.0-m radius curve banked at  $31.0^\circ$  should be driven if the road were frictionless.

#### Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

#### Solution

Starting with

$$\tan \theta = \frac{v^2}{rg},$$

we get

$$v = \sqrt{rg \tan \theta}.$$

Noting that  $\tan 31.0^\circ = 0.609$ , we obtain

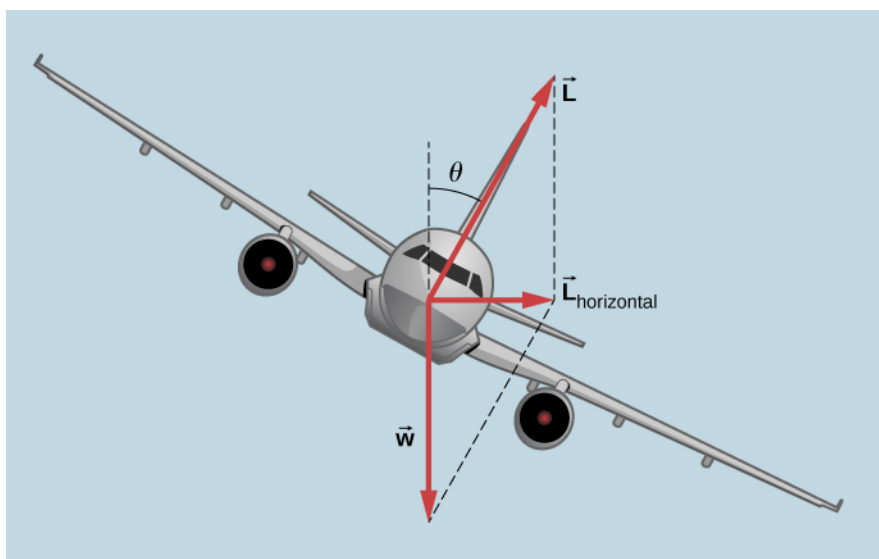
$$v = \sqrt{(100.0 \text{ m})(9.80 \text{ m/s}^2)(0.609)} = 24.4 \text{ m/s}.$$

#### Significance

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Airplanes also make turns by banking. The lift force, due to the force of the air on the wing, acts at right angles to the wing. When the airplane banks, the pilot is obtaining greater lift than necessary for level flight. The vertical component of lift

balances the airplane's weight, and the horizontal component accelerates the plane. The banking angle shown in **Figure 6.23** is given by  $\theta$ . We analyze the forces in the same way we treat the case of the car rounding a banked curve.



**Figure 6.23** In a banked turn, the horizontal component of lift is unbalanced and accelerates the plane. The normal component of lift balances the plane's weight. The banking angle is given by  $\theta$ . Compare the vector diagram with that shown in **Figure 6.22**.



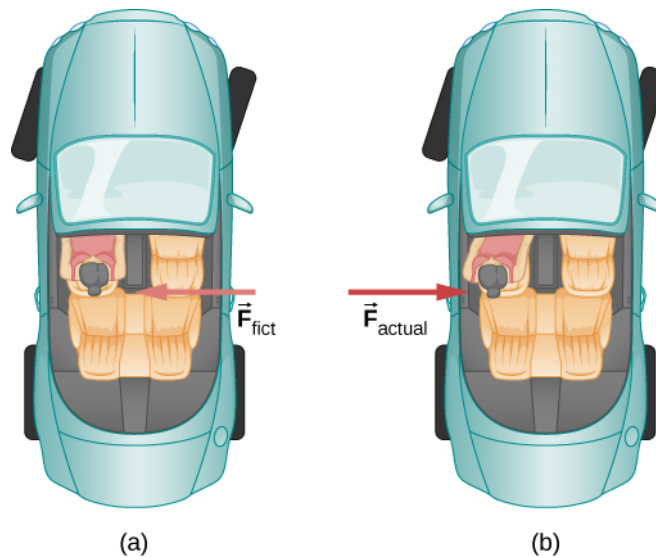
Join the **ladybug** (<https://openstaxcollege.org//21ladybug>) in an exploration of rotational motion. Rotate the merry-go-round to change its angle or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's  $xy$ -position, velocity, and acceleration using vectors or graphs.



A circular motion requires a force, the so-called centripetal force, which is directed to the axis of rotation. This simplified **model of a carousel** (<https://openstaxcollege.org//21carousel>) demonstrates this force.

## Inertial Forces and Noninertial (Accelerated) Frames: The Coriolis Force

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits inertial forces—forces that merely seem to arise from motion, because the observer's frame of reference is accelerating or rotating. When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right (**Figure 6.24**). You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line (recall Newton's first law) but the *car* moves to the right, not that you are experiencing a force from the left.

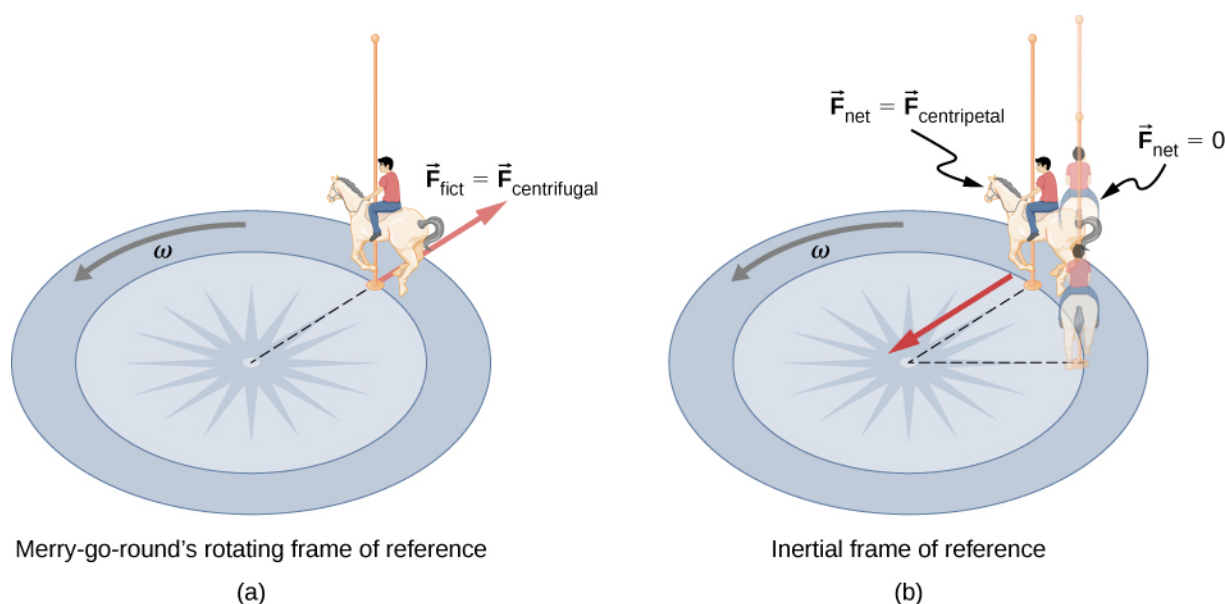


**Figure 6.24** (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is an inertial force arising from the use of the car as a frame of reference. (b) In Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no force to the left on the driver relative to Earth. Instead, there is a force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, whereas a physicist might use Earth. The physicist might make this choice because Earth is nearly an inertial frame of reference, in which all forces have an identifiable physical origin. In such a frame of reference, Newton's laws of motion take the form given in **Newton's Laws of Motion**. The car is a **noninertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is an **inertial force** having no physical origin (it is due purely to the inertia of the passenger, not to some physical cause such as tension, friction, or gravitation). The car, as well as the driver, is actually accelerating to the right. This inertial force is said to be an inertial force because it does not have a physical origin, such as gravity.

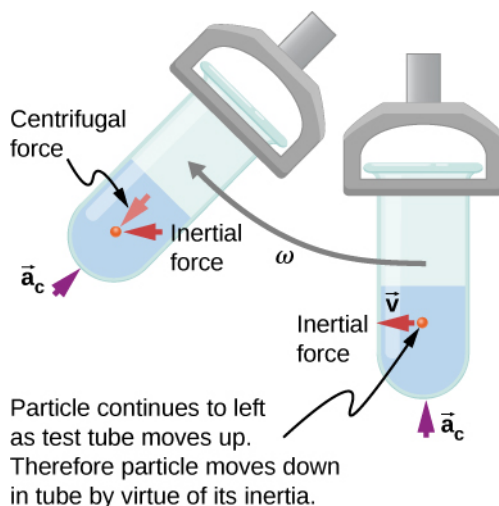
A physicist will choose whatever reference frame is most convenient for the situation being analyzed. There is no problem to a physicist in including inertial forces and Newton's second law, as usual, if that is more convenient, for example, on a merry-go-round or on a rotating planet. Noninertial (accelerated) frames of reference are used when it is useful to do so. Different frames of reference must be considered in discussing the motion of an astronaut in a spacecraft traveling at speeds near the speed of light, as you will appreciate in the study of the special theory of relativity.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round (**Figure 6.25**). You take the merry-go-round to be your frame of reference because you rotate together. When rotating in that noninertial frame of reference, you feel an inertial force that tends to throw you off; this is often referred to as a *centrifugal force* (not to be confused with centripetal force). Centrifugal force is a commonly used term, but it does not actually exist. You must hang on tightly to counteract your inertia (which people often refer to as centrifugal force). In Earth's frame of reference, there is no force trying to throw you off; we emphasize that centrifugal force is a fiction. You must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round, in keeping with Newton's first law. But the force you exert acts toward the center of the circle.



**Figure 6.25** (a) A rider on a merry-go-round feels as if he is being thrown off. This inertial force is sometimes mistakenly called the centrifugal force in an effort to explain the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off (the unshaded rider has  $F_{\text{net}} = 0$  and heads in a straight line). A force,  $F_{\text{centripetal}}$ , is needed to cause a circular path.

This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (**Figure 6.26**). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the inertial force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.

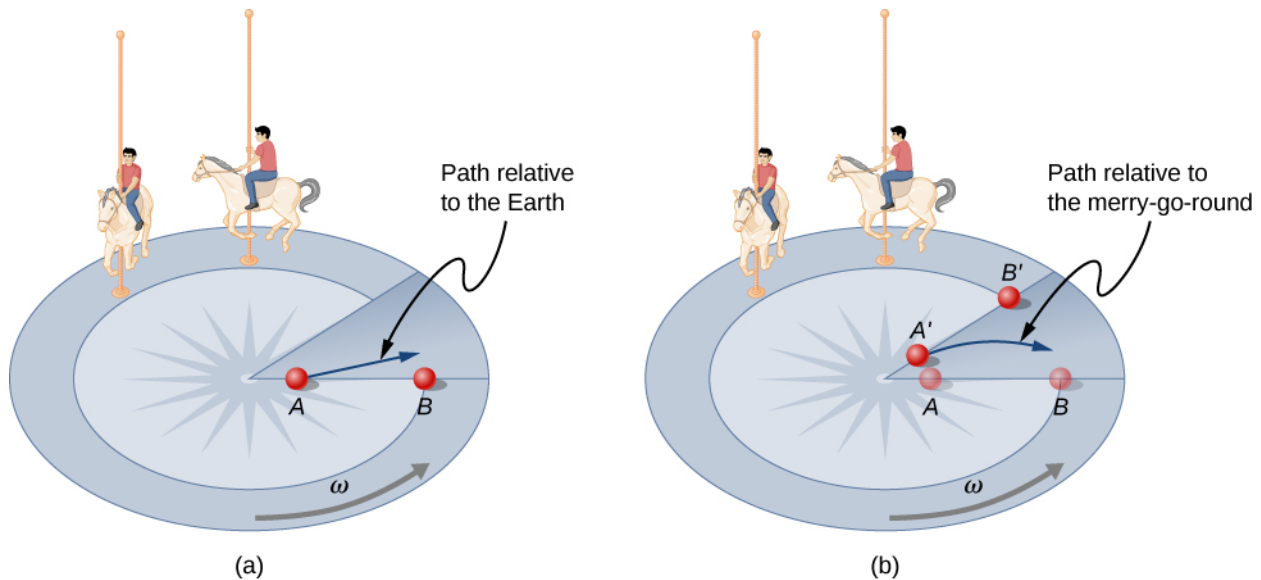


**Figure 6.26** Centrifuges use inertia to perform their task. Particles in the fluid sediment settle out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles come into contact with the test tube walls, which then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a rotating frame of reference. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in **Figure 6.27**? The ball follows a straight path relative to



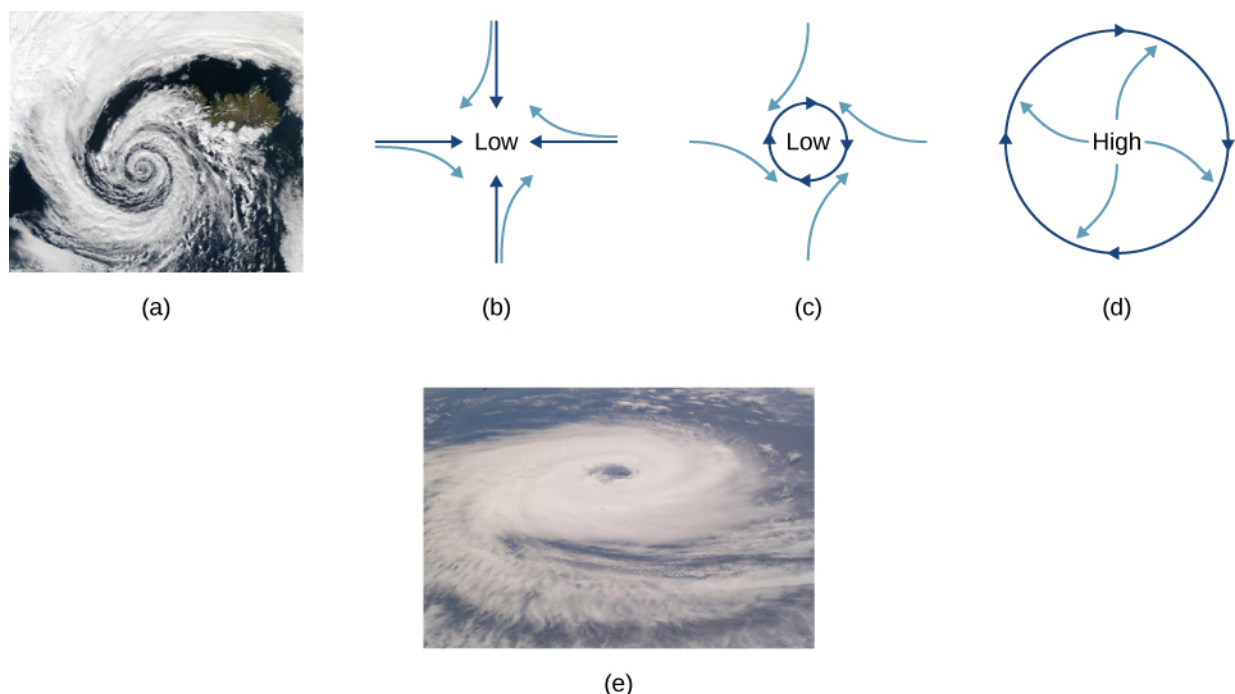
Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using an inertial force, called the **Coriolis force**, which causes the ball to curve to the right. The Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's laws in noninertial frames of reference.



**Figure 6.27** Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point  $B$ , starting at point  $A$ . Both points rotate to the shaded positions ( $A'$  and  $B'$ ) shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in **Figure 6.27**. As on the merry-go-round, any motion in Earth's Northern Hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the Southern Hemisphere; there, the force is to the left. Because Earth's angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the Northern Hemisphere to rotate in the counterclockwise direction, whereas tropical cyclones in the Southern Hemisphere rotate in the clockwise direction. (The terms hurricane, typhoon, and tropical storm are regionally specific names for cyclones, which are storm systems characterized by low pressure centers, strong winds, and heavy rains.) **Figure 6.28** helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the Northern Hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the Southern Hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.



**Figure 6.28** (a) The counterclockwise rotation of this Northern Hemisphere hurricane is a major consequence of the Coriolis force. (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the Southern Hemisphere, leading to tropical cyclones. (credit a and credit e: modifications of work by NASA)

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When noninertial frames are used, inertial forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these inertial forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest in the sense that all forces have origins and explanations.

## 6.4 | Drag Force and Terminal Speed

### Learning Objectives

By the end of the section, you will be able to:

- Express the drag force mathematically
- Describe applications of the drag force
- Define terminal velocity
- Determine an object's terminal velocity given its mass

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion.

### Drag Forces

Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as cyclists, cars, and baseballs not moving